STUDY OF CONJUGATE HEAT TRANSFER IN THE NOZZLE GRID OF A GAS-DYNAMIC LASER

A. S. Boreisho, V. V. Naumov,
V. A. Reisig, A. G. Trofimovich,
and V. V. Lobachev

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The characteristics of the flow and heat transfer in the nozzle grid of a gas-dynamic laser in the presence and absence of cooling of the blades are studied numerically in a conjugate formulation.

One of the main engineering-physics problems in the development of technological setups with a powerful GDL is the construction of the nozzle apparatus — grids of small-scale flat or axisymmetric supersonic nozzles, efficiently freezing the stored vibrational energy and ensuring good optical quality of the flow and adequate pressure to allow discharge into the atmosphere [1-3]. In addition, the gas-dynamic problem of formation of the laser-active medium must be solved simultaneously with heat and strength problems, guaranteeing the working capacity and reliability of the structural elements. As practice shows, successful solution of these problems largely depends on how realistic the mathematical models employed for describing the working process are.

The intensive thermal load on the nozzle apparatus, owing to the high initial temperature (1500-3000°K) and pressure (2-5 MPa) or the working gas, causes the nozzle blades, whose surface temperature in the absence of cooling can become over a short period of time (∿1 sec) comparable to the stagnation temperature of the flow, to overheat. Aside from the degradation of the strength properties of the structure, heating of the walls of the elementary micronozzles comprising the grid has a negative effect on the gas-dynamic flow (the thickness of the boundary layer increases, the nonviscous core of the flow becomes distorted, etc.) and on the kinetics of the relaxation processes (the rate of freezing of the gas drops, losses accompany deactivation in the boundary layer and at the walls, etc.). Ultimately this lowers the operating efficiency of the nozzle grid, which is evaluated based on the relative drop in the useful vibrational energy and gain of the medium [4]. Conversely, when the nozzle walls are intensively cooled (to 300-400°K) additional vibrational energy can be stored in the gas [5], and in addition the optical amplification in the boundary layer can appear even earlier than in the core of the flow [6].

In [4-6] only the stationary states of the flow, when the temperature of the nozzle walls was assumed to be fixed and constant, were examined in the study of the effect of viscous effects on the characteristics of nozzles in the GDL. In reality, however, because of the definite slowness of the thermal relaxation of a solid the temperature of the blade surfaces varies substantially above the blades and in time [7]. The temperature of the walls, affecting the flow of the working mixture along the nozzle, itself depends on the parameters of the gas flow, since the change in the boundary-layer profiles brought about by the heating of the blades as the flow waves over them alters the distribution of heat transfer along the nozzle, which in its turn affects the temperature field in the body of the blades. All this makes it necessary to analyze the working process in the nozzle grid taking into account the nonstationary thermal state of the structural elements, i.e., in a conjugate formulation [8, 9].

This work is concerned with the numerical solution of the conjugate problem of flow and heat transfer of a high-temperature vibrationally nonequilibrium gas in a grid of flat Laval nozzles with the following geometry: the minimum cross section is 0.22 mm high, the degree of expansion equals 40, the subsonic part of the blades is cylindrical and has a diameter

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of 8.9 mm, the supersonic part is 22 mm long and shaped by the method of characteristics so as to obtain nondetached and uniform expansion of the flow, and the output edge is 0.5 mm thick.* Two types of blades were studied: uncooled and water-cooled through an opening 4.5 mm in diameter in the maximum midsection of the blades. The working gas consisted of the mixture 0.1 CO_2–0.87 N_2–0.03H_2O with stagnation parameters T_0 = 2000°K and P_0 = 4 MPa.

The problem formulated mathematically reduces to the solution of the system of boundary layer equations for a laminar flow of a multicomponent vibrationally nonequilibrium gas in a supersonic nozzle in the grid

$$\frac{\partial}{\partial s}(\rho u)+\frac{\partial}{\partial n}(\rho v)=0;$$

$$\rho u\frac{\partial u}{\partial s}+\rho v\frac{\partial u}{\partial n}=-\frac{\partial p}{\partial s}+\frac{\partial}{\partial n}\left(\mu\frac{\partial u}{\partial n}\right);\quad \frac{\partial p}{\partial n}=0;$$

$$\rho u\frac{\partial H}{\partial s}+\rho v\frac{\partial H}{\partial n}=-\frac{\partial q}{\partial n}-u\frac{\partial p}{\partial s}+\mu\left(\frac{\partial u}{\partial n}\right)^2; \tag{1}$$

$$\rho u\frac{\partial E_i}{\partial s}+\rho v\frac{\partial E_i}{\partial n}=\frac{\partial}{\partial n}\left(\lambda_i\frac{\partial T_i}{\partial n}\right)+\rho q_i,\ i=2,\ 3,\ 4;$$

$$pm=\rho RT;\ H=\frac{RT}{m}(3{,}5+0{,}5\xi_{H_2O})+\sum_i\xi_iE_i;\ E_i=\frac{R}{m}g_i\theta_i\left[\exp\left(-\frac{\theta_i}{T_i}\right)-1\right]^{-1};$$

$$m=\sum_i m_i\xi_i;\ q=-\lambda_a\frac{\partial T}{\partial n}-\sum_i\lambda_i\frac{\partial T_i}{\partial n};\ \lambda_i=\rho D_i\frac{dE_i}{dT_i};\ \mathrm{Sc}=0{,}7$$

and the heat-conduction equation for the nozzle blade in the flow

$$\frac{\partial}{\partial x}\left(\lambda\frac{\partial T_{\mathrm{T}}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T_{\mathrm{T}}}{\partial y}\right)+c\rho\frac{\partial T_{\mathrm{b}}}{\partial \tau}+q_v=0. \tag{2}$$

The parameters of the gas at the outer boundary of the boundary layer

$$n=n_e,\ u=u_e(s),\ T=T_e(s),\ E_i=E_{ie}(s),\ p=p(s)$$

were determined by solving the corresponding system of equations for the viscous core of the flow:

$$\frac{d}{ds}(\rho uh)=0;\ \rho u\frac{du}{ds}=-\frac{dp}{ds};\ \frac{dH}{ds}+u\frac{du}{ds}=0;\ u\frac{dE_i}{ds}=q_i,\ i=2,3,4. \tag{3}$$

The conditions of impermeability, sticking, and catalytic activity were employed as the boundary conditions at the nozzle wall:

$$n=0,\ u=v=0,\ T=T_w(s),\ E_2=E_2(T_w),\ E_3=E_3(T_w),\ \frac{\partial E_4}{\partial n}=0.$$

The profiles of the longitudinal velocity u_0, temperature T_0, pressure P_0, and equilibrium values of the vibrational energy E_0 were fixed in the initial section of the boundary layer ($s = s_0$). Initially (τ = 0) the temperature at all points of a blade equals T_0 = 293°K. The condition of heat transfer at the gas-surface interface was given in the form $q_w|_{n-0}=-\lambda_{\mathrm{b}}\frac{\partial T_{\mathrm{b}}}{\partial n}|_{n+0}$. The standard boundary conditions of the third kind were employed in the calculation of the heat transfer in the inner channel of the cooled blade: $\alpha_x(T_w-T_x)=-\lambda_{\mathrm{T}}\frac{\partial T_{\mathrm{T}}}{\partial n}$, where the temperature of the heat transfer agent T_X = 323°K and the heat transfer coefficient $\alpha_x=6\cdot10^4$ W/m^2 are constant. The thermal conductivity of the blade material λ_{T}= 20 W/m•K, while the

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*The profile and dimensions of the trans and supersonic part of the nozzle array for the analysis were taken from the description in [10].

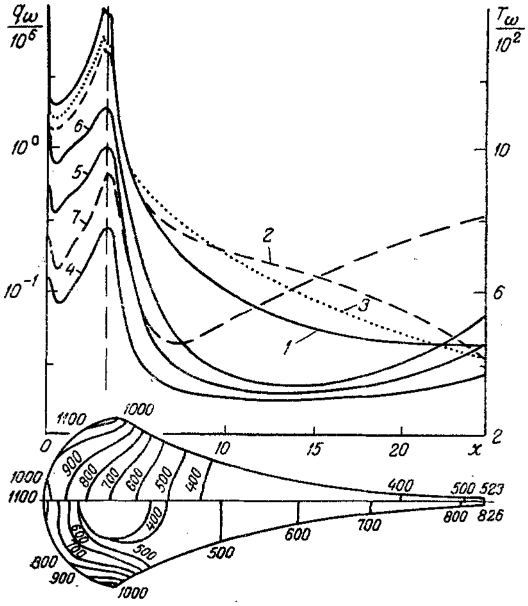

Fig. 1. Temperature field in the transverse section of a blade (a: uncooled, T = 2 sec; b: cooled, τ = 25 sec); the heat flux density distribution (1: isothermal blade, T_w = 300°K, τ = 0 sec, 2: uncooled, τ = 2 sec; 3) cooled, τ = 25 sec) and the temperatures of the surface along the contour of the nozzle (4–6): uncooled, τ = 0.4, 1.2, and 2 sec; 7: cooled, τ = 25 sec). $q_w/10^6$ W/m^2; $T_w/10^2$°K, x, mm.

thermal diffusivity $a_т = 4.4 \cdot 10^{-5}$ m^2/sec. The transport properties of the working mixture were determined using the formulas from the molecular-kinetic theory of gases. The source terms in the kinetic equations, the rate constants of the relaxation processes, and the gain of the standard laser transition P (20) of the 00°1–10°0 band of the CO_2 molecule were described in accordance with the standard "four-temperature" model of vibrational energy transfer in a CO_2 GDL [2].

The system of gas-dynamic and kinetic equations together with the boundary conditions, after the corresponding transformations, were integrated numerically by the method of finite differences [11] using an implicit two-layer scheme of the Crank–Nicholson type for the boundary-layer equations (1) and the standard second-order difference scheme for the equations of the nonviscous core of the flow (3) analogously to [6]. The heat-conduction equation (2) with the initial and boundary conditions were integrated by the method of finite elements [12] using a grid of triangular simplex elements.

The conjugate nature of the problem was taken into account with the help of an iteration procedure for establishing the solution of the nonstationary heat-conduction problem simultaneously with the quasistationary solution of the gas-dynamic problem. The parameters of the boundary layer and the distribution of the heat flux $T_w(s, \tau)$ were determined for a fixed distribution of the temperature of the blade surface $q_w(s, \tau)$. Then the temperature field in the blade $T(x, y, \tau)$ was calculated. Starting from the surface temperature distribution obtained, the characteristics of the flow along the nozzle were calculated again and the conservation of energy was checked under the condition that the temperatures at the interface between the media are equal: the heat flux in the solid body must equal the heat flux from the gas. If a discrepancy existed, the calculation was repeated again. The applicability of this approach is justified by the fact that in this problem the nonstationary nature of the problem is an internal process, determined by the thermal state of the blade, while the characteristic times of the relaxation processes in the gas phase ($\tau_g \sim 10^{-5}$ sec) are much shorter than the thermal relaxation time in the solid ($\tau_s \sim 1$ sec). This is sufficient for the existence of a quasistationary flow regime in the boundary layer (Strouhal's number

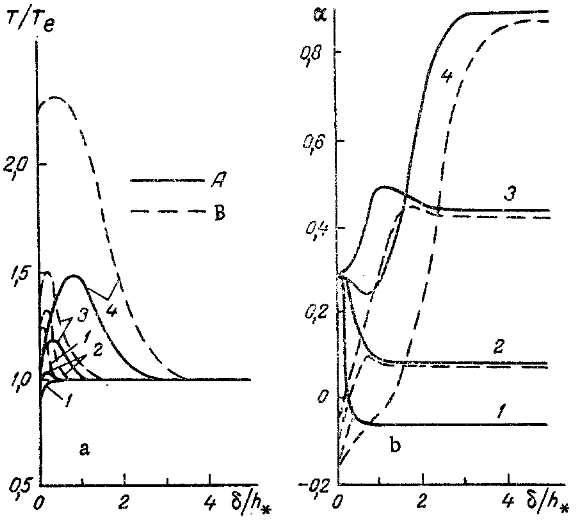

Fig. 2. Temperature profile (a) and gain profile (b) in the boundary layer in the nozzle sections (1-4 — h/h_* = 8, 13, 23, 39): A) isothermal blade, T_w = 300°K, τ = 0 sec; B) cooled bladed, T_w = var, T = 25 sec. α, m^{-1}.

$Sh = \tau'_s/\tau_g \gg 1$) [9].

As a result of the solution of the conjugate problem posed, the distribution of the characteristics of the flow of vibrationally relaxing gas along a nozzle in the grid and the temperature field in the body of the nozzle blade in the presence and absence of cooling was determined, and the time-dependence of these parameters was also found.

The distribution of the heat flux density along the contour of a blade (Fig. 1) is determined by the dynamics of the development of the boundary layer and is typical for flow in a supersonic nozzle. The existence of characteristic maxima in the heat transfer in the vicinity of the noze point and near the critical section leads to the establishment of corresponding maxima of the temperature of the blade surface. The total thermal load on the blade equals 70 kW/m, 85% of which falls in the sub- and transonic regions of the nozzle.

A significant redistribution of the heat-transfer coefficients along the blade contour, caused by the well-known [8] effect of the variability of the temperature of the surface along the longitudinal coordinate and in time, was observed: the heat transfer coefficients are higher with an increasing temperature head and lower with a decreasing temperature head than in the case of an isothermal and stationary thermal state of the surface. Therefore the use of heat-transfer coefficients determined for the case $dT_w/ds = 0$ in the calculation of the nonstationary thermal state of a blade leads to some decrease in the temperature of the surface on sections where $dT_w/ds < 0$.

Comparison of the results of the calculation of the temperature field of a blade in the separate and conjugate formulation showed that the tail part of the blade is most sensitive to taking into account the conjugate nature of the problem. Thus for an uncooled structure the total heat flux flowing per unit time into the tail part of the blade up to the moment T = 2 sec is 60% higher than the value obtained in the nonconjugate calculation. The heating of the blade body is also correspondingly accelerated: the heating time for the output edge up to T_w = 500°K neglecting the conjugate formulation equals 2 sec, whereas taking into account the change in the thermal regime in the conjugate calculation this time drops to 1.5 sec. For a cooled blade, after a stationary thermal state is established (after 25-30 sec) the temperature of the output edge turns out to be 170°K higher when the conjugate nature of the problem is taken into account. At the same time in the vicinity of the frontal point there is virtually no difference between the calculations performed in the separate and conjugate formulations.

The difference in the behavior of the components of the heat flux is interesting: the convective part, determined by the transfer of the translational and rotational energy of the molecules, decreases as T_w increases, whereas the diffusion part, determined by the transfer of vibrational energy of the molecules, depends in a complicated manner on the structure of the flow, the temperature of the surface, and the catalytic activity of the material of the wall

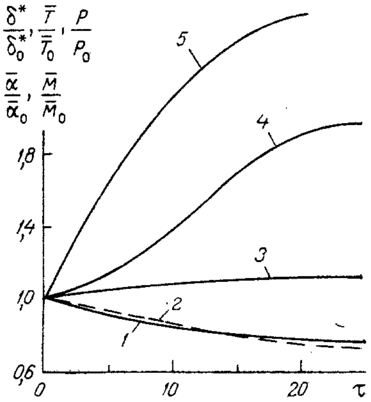

Fig. 3. Relative change in the parameters of the working medium at the cutoff of an uncooled nozzle in the grid as a function of time: 1 — $\overline{\alpha}$; 2 — $\overline{M}$; 3 — P; 4 — $\overline{T}$; 5 — δ^*. τ, sec.

relative to different vibrationally excited components. Because there is no interaction with the wall the distribution of the vibrationally excited N_2 molecules in the boundary layer is practically frozen and has virtually no effect on the change in the enthalpy of the gas. Because of the heterogeneous deactivation of the vibrationally excited CO_2 molecules, however, the enthalpy of the gas near the surface increases, and ultimately the diffusion part of the heat flow also decreases as T_w increases.

The character of the deformation of the boundary layer profiles in the supersonic region of the nozzle indicates that the heating of the blade significantly affects not only the dynamic and thermal but also the inversion characteristics of the flow (Fig. 2). In the nonviscous core of the flow rapid ($dT/d\tau \sim 10^9$ deg/sec) adiabatic cooling of the working mixture brings about freezing of the upper laser level (the parameter $P_0 h_*/2\mathrm{tg}\,\theta = 6$ atm mm) and continuous growth of the optical gain as the translational temperature of the gas decreases, whereas the formation of the active medium in the boundary layer is affected by the heating of the gas owing to the viscous friction, the cooling of the gas owing to heat conduction, and the catalytic activity of the wall. At sufficiently low surface temperatures (initially $T_w/T_0 \leqslant 0.3$), irrespective of the state of the gas at the outer boundary of the boundary layer, a population inversion arises owing to the heterogeneous depletion of the symmetric and deformation modes of CO_2 ($T_{1,2}$) with simultaneous population of the antisymmetric mode CO_2 (T_3) by means of reasonant transfer of energy from the vibrational mode N_2 (T_4). The remaining gas-phase vibrational relaxation processes, according to the analysis of the characteristic Damkeler numbers ($Da \leqslant 1$), are ineffective. The profile of T_3, like that of T_4, is practically frozen in the boundary layer and hardly reacts at all to a change in T_w (although near the wall as T_w increases the difference between T_3 and T_4 decreases). Conversely, the T_2 profile, owing to the strong coupling between the translational degrees of freedom and the symmetric modes of CO_2, follows almost synchronously the heating of the gas in the boundary layer as T_w increases. It is these features of the distribution of the nonequilibrium parameters of the medium that are responsible for the nonmonotonic change in the profiles of the gain α, as shown in Fig. 2. We note that taking into account the nonisothermal state of the blades gives viscous losses of 12.5% in the gain at the nozzle cutoff instead of 5.8% for an isothermal blade. Neglecting the conjugate nature of the problem also introduces an error into the determination of the integral characteristics of the flow of vibrationally nonequilibrium gas in the nozzle, since the thicknesses of the boundary layer δ^* are too small, while the heat fluxes q_w are too high (the latter could also be linked with the fact that the thermal conductivity of the gas is too high near the wall, which happens in the calculation neglecting the heating of the blade).

The dependence of the mean-mass parameters of the working medium at the nozzle cutoff on the operating time of the nozzle grid with an uncooled construction is of greatest practical interest (Fig. 3). As the calculations show, initially (up to 1-1.5 sec) the change in the parameters compared with the values calculated in the approximated of isothermal blades, is small. However over the time of the complete heating of the blade (20-25 sec) up to the stagnation temperature of the gas (i.e., for a temperature factor $T_W/T_0 = 1$) the parameters of the supersonic flow change substantially: Mach's number decreases by 23%, the static temperature increases by 100%, and the gain drops by 22%. Taking into account the nonisothermal state of the blade has a qualitatively analogous effect also in the calculation of the parameters of a flow in a nozzle grid cooled in a stationary manner. Of course, the degree of this effect can be substantially reduced by selecting a more effective system for cooling the blade.

Thus the numerical studies performed demonstrated the desirability of analyzing the flow and heat transfer in the nozzle array of a GDL taking into account the interaction of the blades in the flow and the gas flow, especially when the structure is strongly heated. This is explained by the abundance of parameters determining the solution of the problem and the existence of a number of factors intensifying the effect of the conjugate nature of the problem [8]: laminarity of the flow, decreasing temperature head, high negative pressure gradients (under the condition studied the characteristic Reynolds number $\mathrm{Re}_0 = \rho_0\sqrt{2H_0}h_*/\mu_0 = 2\cdot 10^4$, the conjugation parameter $S = \mathrm{Pr}\sqrt{\mathrm{Re}_0}\lambda_{r0}/\lambda_{T0} = 0.6$). Of course, because of the approximate nature of the mathematical model employed for the working process, associated with some uncertainty in the coefficients of the kinetic processes, the catalytic activity of the material of the walls, etc., the conjugate formulation of the problem does not so much raise the accuracy of the calculation as it illustrates the degree to which the thermal state of the nozzle apparatus affects the formulation of the active medium of the GDL. Nevertheless it could be very important to take this effect into account in order to choose the optimal structural layout of the nozzle grid in building a continuous GDL.

NOTATION

T, temperature; P, pressure; ρ, density; s, coordinate along the generatrix of the nozzle contour; n, coordinate along the normal to the surface of the nozzle; u and v, longitudinal and transverse components of the velocity vector; μ, λ, and D, coefficients of viscosity, thermal conductivity, and diffusion; R, universal gas constant; m, molecular weight of the mixture; H, specific enthalpy of the gas; ξ_i, molar fraction of the components of the mixture; E_i, specific energy of the vibrational mode; θ_i and T_i, characteristic and local vibrational temperatures; g_i, degeneracy of the vibrations; α, optical gain (at a wavelength of 10.6 μm); δ^*, thickness of the boundary layer; h, height of the transverse section of the nozzle; θ, half-angle of expansion of the supersonic part of the nozzle; x, y, coordinate system fixed to the transverse section of the blade; c, the specific heat capacity; τ, time; q_v, source term, characterizing the intensity of heat transfer in the blade; q_i, source term characterizing the intensity of vibrational energy transfer in the gas; q_t, heat flux density from the gas into the nozzle wall; Re, Reynolds number; Pr, Prandtl's number; Sc, Shmidt's number, Sh, Strouhal's number, Da, Damkeler's number. Indices: 0, stagnation parameters; *, critical section; e, core of the flow; g, gas; b, body, w, blade surface; a, translational-vibrational degrees of freedom; i = 1, 2, 3, symmetric, deformation, and antisymmetric vibrational modes of CO_2; i = 4, vibrational mode of N_2.

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PLANE TURBULENT WAKE WITH ZERO EXCESS MOMENTUM

Yu. M. Dmitrenko, I. I. Kovalev,
N. N. Luchko, and P. Ya. Cherepanov

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A plane momentumless wake is investigated on the basis of numerical computations by the $\overline{u_i u_j}-\varepsilon$ model of turbulence and an aerodynamic experiment.

1. Introduction. The turbulent wake behind a flat body constantly attracts the attention of researchers not only because of the numerous technical applications but also as one of the simplest modifications of free shear flow convenient for the confirmation of moder semiempirical turbulence models. In a number of engineering installations, for instance, in atomizers, mixing units, flame stabilizers, jet flaps, etc., the wakes are formed under conditions of jet efflux from holes located in the root area of streamlined bodies. Here, dependent on the velocity of jet efflux, coflows with a different magnitude of the excess mementum are realized

$$I = \rho \int_0^\infty [U_1(U_1 - U_\infty) + \overline{u}_1^2 - \overline{u}_2^2]\, dx_2. \tag{1}$$

Of special interest is the particular case of such a coflow when the excess momentum equals zero, which corresponds to the condition of equality of the hydrodynamic drag of the streamlined body and the reactive thrust produced by the jet.

In contrast to axisymmetric flow, the number of papers devoted to investigation of a plane wake with zero excess momentum is quite limited. The results of a self-similar analysis for I = 0 is contained in [1-4]. The self-similar analysis permits expression of the exponent in the damping laws of the wake characteristics in terms of one unknown parameter whose magnitude is either determined from experiment [2, 3] or calculated analytically by using certain additional assumptions (for instance, about the constancy of the turbulent viscosity across the wake [1, 4]). A plane coflow with zero excess momentum was investigated experimentally in just one paper [5]. Average velocity profiles were measured in comparative detail therein while the fluctuating characteristics are represented by turbulent energy and tangential stress profiles in just one section. In this connection, available experimental data are inadequate for a critical estimate of the results of self-similar analysis.

Qualitatively, the results of [1-5] are in agreement and show that in principle, the differences in the evolution of the wake characteristics for I = 0 and I ≠ 0 are inherent not only to axisymmetric but also plane wakes. In particular, the defect in the mean velocity damps out in proportion to the kinetic energy of the fluctuations. At a certain distance from the streamlined body the turbulent energy generation becomes negligibly small as compared with its dissipation rate and the turbulent Reynolds number decreases downstream (in contrast to the case I ≠ 0).

As regards the quantitative estimates, the results of the above-mentioned papers differs substantially. For example, the following estimates $n_u = -0.75$ [1, 5], $n_u = -1.11$ [4], $n_u = -1.5$ [2, 3] are obtained for the exponent n_u in the self-similar law of velocity defect damping. Just as radically different are the exponents for the other characteristics also. Taking account of the known constraints of the self-similarity analysis methods and the contradictions in the quantitative results obtained by using them, as well as the absence of detailed experimental data, results of an aerodynamic experiment and a numerical computation utilizing a multiparametric differential model of turbulence are used to determine the regularities of the evolution of the characteristics of a plane turbulent wake with zero

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A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 5, pp. 743-751, May, 1987. Original article submitted March 26, 1986.

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